

CIRAD

# DYNMOD

**A spreadsheet interface for demographic  
projections of tropical livestock  
populations – User's Manual**

**M. Lesnoff**

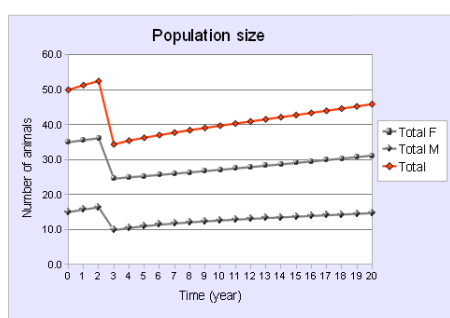


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### Forewords

This document, intended for educational use, is a handbook to be used by researchers, engineers, technicians or students dealing with demography of tropical ruminant livestock (cattle, small ruminants and camels).

Choices and opinions expressed in the text are the only responsibility of the author.

### Acknowledgements

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# 1 Introduction

Livestock populations represent a major economic value for many tropical developing countries. Tools for evaluating impacts of development projects or shocks (like drought or outbreaks) on livestock dynamics and productions are helpful for decision making in livestock management.

The present document is the user's manual of DYNMOD, which is a simple herd growth model interface for ruminant livestock populations. At its origin, DYNMOD has been designed for pedagogical objectives. The underlying demographic model is therefore very simplified (much more elaborated and complex research-orientated models can be designed). Several experiences showed, however, that it also can help non-specialists to implement fast and crude ex ante or ex post demographic diagnostics, in various applications as for example livestock population management, herd production estimation or exploration of scenarios in development projects.

Based on simplified demographic equations, DYNMOD simulates the dynamics of the population size and the number of animals produced per year. DYNMOD also calculates live weights, meat production and secondary productions (milk, skin and hides, manure) at population level, as well financial outputs that can be used in more integrated financial calculations (e.g. benefit-costs ratios or internal return rates). Finally, DYNMOD provides crude estimates of the population feeding requirements in dry mater.

DYNMOD predicts what would happen under some hypothetical scenarios, not what will happen in the real world. In that sense, DYNMOD is more a projection model than a prediction model ([1]). In a simulation, the user has to judge by itself of the relevancy of the scenario considered.

A first version of DYNMOD was initially developed jointly by CIRAD and ILRI (International Livestock Research Institute) in 2007. New versions have then been successively designed by CIRAD. The present version of DYNMOD consists in three spreadsheet modules:

- STEADY1: simulation of the 1-year population production assuming a demographic steady state – with constant sex-by-age structure and growth rate;
- STEADY2: simulation of the 1-year population production assuming a demographic steady state – with constant adult population size and null growth rate;

- PROJ: simulation of the population dynamics and production of a over a period of time in a possibly variable environment (the period can last from 1 to 20 years).

The three modules are based on the same demographic model but represent different hypotheses.

The main parameters of DYNMOD are the *demographic rates* (reproduction, mortality, etc.). Theoretical concepts on these demographic rates (e.g. notions on probabilities, hazard rates, competing risks and interferences between rates) are not described in the present handbook. They are regarded as prerequisites. Before using DYNMOD, we recommend the reader to refer to documents detailing these concepts. For instance, two of such specialized documents ([2],[3]) are available at <http://www.quae.com> or <http://livtools.cirad.fr>, respectively.

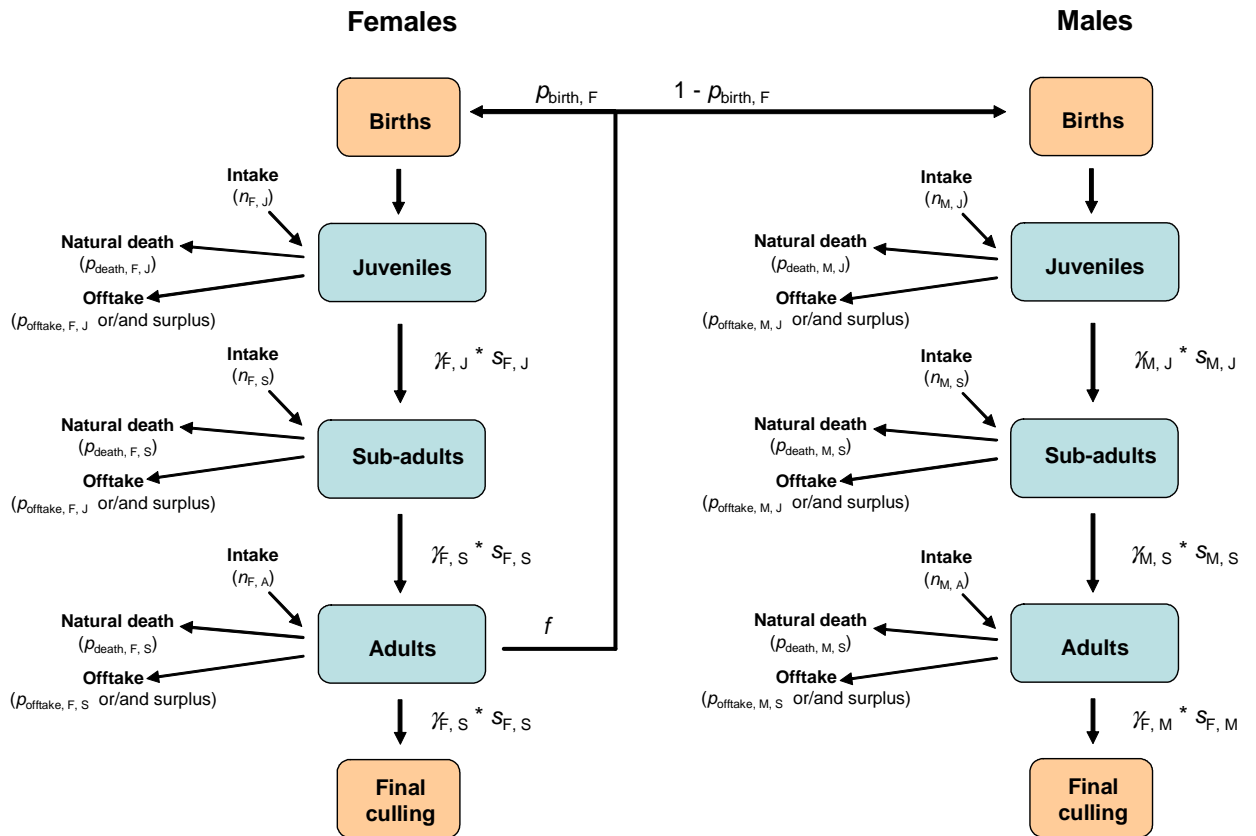
All materials related to DYNMOD can be downloaded at <http://livtools.cirad.fr>.

# 2 General characteristics

## 2.1 Model structure

In DYNMOD, livestock populations are divided by sex (female and male) and age class (Figure 1). In each sex, three age classes and their durations must be defined: juveniles, sub-adults and adults.

Durations of the age classes can depend on biological or farming management traits or on the way that the user needs to present the results (see Chapter 4 for an example). In the model, only the adult females are assumed to be reproductive.



**Figure 1:** Simplified animal life cycle defining the structure of the demographic model in DYNMOD.

## 2.2 Dynamics and parameters

Herd dynamics result from births, mortality (i.e., deaths due to other causes than slaughtering; referred as natural deaths in this handbook), offtake (slaughtering, sales, gifts, departures due to loans, etc.) and intake (purchases, gifts, arrivals of loans, etc.).

In DYNMOD, births are determined by two reproduction rates of adult females: the parturition and net prolificacy rates. Newborn are distributed between females and males following the “proportion of females at birth”. Reproduction is assumed to be distributed all over the year. This is a realistic assumption for tropical livestock populations in extensive farming systems, but generally not for wildlife populations with birth-pulse reproduction ([1]).

As a remark, DYNMOD can be run without gender. For instance, a female population can be simulated by dividing the parturition rate by two and setting the proportion of females at birth to 100%.

In each age class, animals can survive or be removed from the population by natural death or offtake. Natural deaths are simulated by applying mortality rates to animals living in the population. Offtake are simulated as follows:

- Offtake rates are applied to animals growing in the age classes;
- Additionally, all the animals surviving until the end of the adult age class are automatically culled (this corresponds to the final culling) and considered as offtake;
- When a constant or maximum size is set for an age class, all the surplus is removed of the population and considered as offtake.

Being difficult to represent simply in herd growth spreadsheets, intakes (purchases, loans, gift, etc.) are frequently neglected. In DYNMOD, two ways can be used to take into account for intakes, jointly or separately. The first way is to include them into the offtake rates. In that case, intakes are not explicitly represented: offtake rates correspond to “net offtake rates”, representing balances “offtake minus intake”. This is the case in STEADY1 and STEADY2. The second way (only available in PROJ) is to define a number (instead of a rate) of animals imported by sex and age class, for instance to represent a major restocking of animals after a shock in the population.

Additional parameters can be specified in DYNMOD: live weight and financial value of animals, milk/wool/manure productions and feed requirements.

## 2.3 Type of model and hypotheses

### 2.3.1 Generalities

The mathematical model used in DYNMOD is deterministic (i.e., not random). For simplification, no seasonal variations in the parameters are represented although

it is well-known that demographic rates of tropical livestock can have seasonality. Demographic rates are entered on an annual basis. Nevertheless, the time-interval used in the calculations is the month, for removing the competing risks problem between the different demographic events (Chapter 5).

DYNMOD does not constraint the parameters – for instance, parameters are not population-density dependent or functions of resources (feed, water, etc.) or of economic variables (markets' offer/demand, animals' prices, etc.). Nevertheless, for preventing unrealistic growth in projections, a maximum size for the livestock population can be defined in interface PROJ, the surplus being automatically removed from the population and considered as offtake.

Details on equations and parameters of DYNMOD are presented in Chapter 5.

### 2.3.2 STEADY1 and PROJ

In demographic models, when reproduction and survival rates are constant over time, the population dynamics show a transient phase of fluctuations and then converge to a steady state ([1]). This steady state is characterized by a stable sex-by-age structure and a stable growth rate, depending only on the demographic rates and not on the initial population structure.

Based on the entered demographic rates and the total population size  $N$ , STEADY1 calculates the sex-by-age structure, the annual growth rate and the annual production corresponding to the steady state population.

The steady state population can also be calculated using PROJ by repeating the same parameters values over a sufficiently long period. The steady state population can then be observed at the last year of the simulation.

PROJ was essentially designed to represent inter-annual variability (for example due to environmental factors) by varying “at hand” the values of the parameters of the years simulated.

### 2.3.3 STEADY2

STEADY2 is a variant of the steady-state approach. The number of adult females and males is assumed to be constant. The surplus due to natural population production is removed each month. Keeping constant the number of adult females obliged the population to have a null growth rate, i.e. a constant size in all age classes. The corresponding steady-state numbers of juveniles and sub-adults are automatically calculated based on the demographic rates and the number of adult females.



# 3 DYNMOD interface

## 3.1 Module STEADY1

STEADY1 (Figure 2) is available in file `dynmod_steady1.xlsx`. The interface is composed of two parts: “Parameters” and “Results”.

### 3.1.1 Part “Parameters”

In part “Parameters”, cells used for data entry are in white. Entered data appear in blue characters. Outputs are either in grey or black characters.

The entered parameters values are automatically multiplied by the coefficients specified in columns “Coef.” (by default, set to 1). This coefficient was designed to facilitate sensitivity analyses. Columns “Actual” correspond to the final values (= entered values \* Coef) used for the calculations.

Part “Parameters” is composed of the following five sections.

#### *Age class duration*

- Durations of the age classes (month). After entering the durations, STEADY1 automatically calculates the exact ages (year) delimiting the age classes.

#### *Population*

- Total number of animals in the population at beginning of the year (initial total population size  $N$ ).

#### *Demography*

- Reproduction:
  - Parturition rate per year  $h_{\text{par}}$  (average number of parturitions per year and reproductive female present the whole year in the population). This parturition rate only applies to adult females; other age classes are not reproductive;
  - Net prolificacy rate (average number of offspring born alive per parturition);
  - Proportion of females at birth (average proportion of females within the offspring born alive);
- Mortality:

- Mortality probability  $P_{\text{dea},c}$  that would be observed if no offtake occurred in the population;
- Offtake:
  - Offtake (or net offtake) probability  $P_{\text{off}}$ .

Entered probabilities  $P_{\text{dea},c}$  and  $P_{\text{off}}$  correspond to the entire age class if the age class duration <12 months, or to a 1-year period if the age class duration ≥12 months. Probability  $P_{\text{dea},c}$  quantifies an *intrinsic mortality*, i.e. the mortality that would be observed assuming no other causes of removals ([2],[3] and Chapter 5).

### *Production*

- Milk production (offtake by farmers):
  - Quantity of milk offtake per lactation when the lactation is milked (litre) (in traditional systems, the situation is frequent where farmers does not milk all the lactations);
  - Percentage of lactations milked, within all the lactations occurred in the year;
- Live weight (kg) of the animal at the beginning of the age class (STEADY1 automatically calculates the average live weights by age class, used to calculate live weights at population level);
- Carcass yield (%), used for calculating the meat production;
- Financial value per animal:
  - Per offtake;
  - Per intake;
- Skin and hides production (kg) per offtake;
- Wool production (kg) per animal and year;
- Manure production (kg) per animal and year.

### *Feeding*

- Daily requirements of dry matter (%) per kg of live weight.

### **3.1.2 Part “Results”**

Part “Results” is automatically updated when parameters are modified. This part is composed of the following four sections.

### *Population*

- Number of animals at beginning of the year;
- Number of animals at end of the year;
- Average number of animals over the year.

### *Mortality*

- Number of natural deaths in the year  $m_{\text{death}}$ ;
- Mortality rate defined as  $m_{\text{death}}$  / number of animals at beginning of the year;
- Mortality rate defined as  $m_{\text{death}}$  / average number of animals in the year.

### *Production*

- Offtake:
  - Number of offtake (or net offtake) in the year  $m_{\text{offtake}}$ ;
  - Offtake (or net offtake) rate defined as  $m_{\text{offtake}}$  / number of animals at beginning of the year;
  - Offtake (or net offtake) rate defined as  $m_{\text{offtake}}$  / average number of animals in the year;
- Stock variation + Offtake:
  - Same as for offtake but after adding the stock variation (difference between the population sizes at end and beginning of the year);
- Live weight (kg), meat (kg) and financial equivalents of the:
  - Average number of animals over the year;
  - Number of offtake (or net offtake) in the year;
  - Stock variation + Number of offtake (or net offtake) in the year;
- Milk production (litre) (offtake by farmers) :
  - Average production per reproductive female and year;
  - Total production in the year;
- Skin and hides production (kg) in the year;
- Wool production (kg) in the year;

- Manure production (kg) in the year;
- Productivity measures:
  - Average number of sub-adults produced per reproductive female and year;
  - Average number of adults produced per reproductive female and year.

### *Feeding*

- Dry matter requirements (kg) in the year.

## **3.2 Module STEADY2**

STEADY2 (Figure 3) is available in the file `dynmod_steady2.xlsx`. The interface has the same structure and parameters as STEADY1, except that the (constant) numbers of adult females and males have to be entered, and that no offtake rates need to be entered for adults.

Adult offtake (numbers and rates) are calculated by the model. They correspond to the surplus of adults (compared to the specified constant adult population sizes) which is removed each month of the population. The population growth rate is automatically set to 0.

STEADY1					Parameters					Results				
Age classes					Population					Production				
Duration (month)		Exact age (year)			Size		Structure			Live weight (kg/animal at beginning of age group)				
		from to			Initial		Global Intra-sex			Ref.		Coef.		Actual
Female	Juvenile	12	0.0	1.0	Female	J	4.5	9%	13%	Female	J	20	1.00	20
	Sub-adult	36	1.0	4.0		S	10.3	21%	30%		S	50	1.00	50
	Adult	132	4.0	15.0		A	19.2	38%	57%		A	250	1.00	250
Male	Juvenile	12	0.0	1.0	Male	J	4.2	8%	26%	Male	J	20	1.00	20
	Sub-adult	36	1.0	4.0		S	7.3	15%	46%		S	70	1.00	70
	Adult	72	4.0	10.0		A	4.5	9%	28%		A	300	1.00	300
Demography					Feeding					Meat				
Reproduction					Dry matter requirements (% of kg LW/day)					Financial value (/animal)				
Parturition rate (/year)					Ref. Coef. Actual					Carcass yield (%)				
Net prolificacy rate					F 33.9 68% 100%					Female J 40,000 1.00 40,000				
% of female at birth					M 16.1 32% 100%					S 90,000 1.00 90,000				
					T 50.0 100%					A 150,000 1.00 150,000				
										Male J 40,000 1.00 40,000				
										S 110,000 1.00 110,000				
										A 200,000 1.00 200,000				
										Male J 40,000 1.00 40,000				
										S 110,000 1.00 110,000				
										A 200,000 1.00 200,000				
										Milk				
										Offtake per lactation milked (litre)				
										170 1.00 170				
										% lactations milked				
										80% 1.00 0.8				
										Offtake per lactation (litre)				
										136 136				
										Skin and hides (kg/animal)				
										Female J 0.0 1.00 0.0				
										S 1.0 1.00 1.0				
										A 3.0 1.00 3.0				
										Male J 0.0 1.00 0.0				
										S 1.0 1.00 1.0				
										A 3.0 1.00 3.0				
										Wool (kg/animal)				
										J 1.00 0.0				
										S 1.00 0.0				
										A 1.00 0.0				
										Manure (kg/animal/day)				
										J 0.5 1.00 0.5				
										S 0.8 1.00 0.8				
										A 1.5 1.00 1.5				
										Results				
										Population				
										Size				
										Ini				
										End				
										Avg				
										Number				
										Nb./size				
										Nb./avg size				
										Female J 4.5 4.5 4.5				
										S 10.3 10.4 10.4				
										A 19.2 19.5 19.3				
										Male J 4.2 4.3 4.3				
										S 7.3 7.5 7.4				
										A 4.5 4.5 4.5				
										Total F 33.9 34.4 34.2				
										M 16.1 16.3 16.2				
										T 50.0 50.7 50.4				
										Mortality				
										Death				
										Number				
										Nb./size				
										Nb./avg size				
										Female J 4.5 4.5 4.5				
										S 10.3 10.4 10.4				
										A 19.2 19.5 19.3				
										Male J 4.2 4.3 4.3				
										S 7.3 7.5 7.4				
										A 4.5 4.5 4.5				
										Total F 33.9 34.4 34.2				
										M 16.1 16.3 16.2				
										T 50.0 50.7 50.4				
										Population				
										Offtake				
										Stock variation + Offtake				
										Number				
										Nb./size				
										Nb./avg size				
										Female J 0.0 0.0% 0.0%				
										S 0.5 5.3% 5.2%				
										A 2.1 10.8% 10.7%				
										Male J 0.5 11.3% 11.2%				
										S 1.7 22.8% 22.7%				
										A 1.4 30.5% 30.3%				
										Total F 2.6 7.7% 7.7%				
										M 3.5 21.9% 21.7%				
										T 6.1 12.3% 12.2%				
										Live weight equivalent (kg)				
										Avg liv. stock 9,453				
										Offtake 1,340				
										SV + Offtake 1,471				
										Milk (litre)				
										Avg per reprod. fem. 68				
										Total 1,314				
										Skin & hides (kg)				
										13				
										Meat equivalent (kg)				
										Avg liv. stock 4,443				
										Offtake 630				
										SV + Offtake 691				
										Wool (kg)				
										0				
										Manure (kg)				
										19,828				
										Financial equivalent				
										Avg liv. stock 5,895,147				
										Offtake 835,778				
										SV + Offtake 917,376				
										Productivity measures				
										Nb. new sub-adult/adult f. 44%				
										Nb. new adult/adult f. 37%				
										Feeding				
										Feed requirements (kg)				
										Dry matter 86,261				

Figure 2: Interface STEADY1 of DYNAMOD.

STEADY2					Parameters					Results						
Age classes					Population			Production		Population		Mortality				
Duration (month)		Exact age (year)			Size		Structure	Live weight (kg/animal at beginning of age group)			Size		Death			
		from	to		Initial	Global	Intra-sex	Ref.	Coef.	Actual	Ini	End	Avg	Number	Nb./size	Nb./avg size
Female	Juvenile	12	0.0	1.0	J	4.7	9%	20	1.00	20	J	4.7	4.7	0.6	12.6%	12.6%
	Sub-adult	36	1.0	4.0	S	11.3	22%	50	1.00	50	S	11.3	11.3	0.6	5.1%	5.1%
	Adult	132	4.0	15.0	A	20.0	40%	250	1.00	250	A	20.0	20.0	0.6	3.0%	3.0%
Male	Juvenile	12	0.0	1.0	J	4.5	9%	20	1.00	20	J	4.5	4.5	0.6	12.5%	12.5%
	Sub-adult	36	1.0	4.0	S	8.0	16%	70	1.00	70	S	8.0	8.0	0.4	5.1%	5.1%
	Adult	72	4.0	10.0	A	2.0	4%	300	1.00	300	A	2.0	2.0	0.1	3.0%	3.0%
Demography					Total			Meat			Total			Total		
Reproduction		Ref.	Coef.	Actual	F	M	T	Carcass yield (%)			F	M	T	Stock variation + Offtake		
Parturition rate (/year)		0.50	1.00	0.50				47%						Number		
Net prolificacy rate		1.00	1.00	1.00				Financial value (/animal)						Nb./size		
% of female at birth		50%	1.00	50%				Female						Nb./avg size		
								J						Number		
								S						Nb./size		
								A						Nb./avg size		
								40,000						0.0		
								90,000						0.0%		
								150,000						0.0%		
								150,000						0.0%		
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								40,000						0.0%		
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								200,000						0.0%		
								40,000						0.0%		
								110,000						0.0%		
								200,000						0		

Figure 3: Interface STEADY2 of DYNMOD.

### 3.3 Module PROJ

PROJ is available in the file dynmod\_proj.xlsx. The interface is composed of three parts: “Parameters” (Figure 4 to Figure 6), “Results” (Figure 7 to Figure 9) and “Graphics” (Figure 10). In PROJ:

- Calculations are carried out over a period that can last from 1 to 20 years;
- Intakes (in number of animals) can be entered. In the calculations, intakes are assumed to enter in the population at the middle of the year.

Parameters have to be entered for each year of the simulated period. The duration of the period is defined in the cell “Number of years of projection”. The initial population is defined by its size  $N$  (total number of animals) and its sex-by-age structure (%). PROJ automatically calculates the initial sizes by sex and age class.

When a maximal size (for one or more sex-by-age class) is entered, the initial size used for the projection is the one provided in the column “Initial c”. At the end of each month, PROJ calculates the surplus compared to the specified maximum sizes. When positive, the surplus is removed from the population and added to the offtake (and inversely if the surplus is negative).

Results are displayed for each year of the simulated period. Averages or totals over the period are also provided. When relevant, PROJ calculates the percentage of variation of the output from year  $t$  to year  $t+1$ .

Part “Graphics” is composed of two graphs representing to the population size dynamics and annual growth rate.

PROJ									
Parameters					Parameters				
Age classes					Population				
Duration (month)		Exact age (year)			Size		Structure		Projection
		from	to		Initial	Max.	Initial c	Global	Nb. years of projection
Female	Juvenile	12	0.0	1.0	J	4.5	4.5	9%	20
	Sub-adult	36	1.0	4.0	S	10.0	10.0	20%	
	Adult	132	4.0	15.0	A	20.5	20.5	41%	
Male	Juvenile	12	0.0	1.0	J	4.0	4.0	8%	Type
	Sub-adult	36	1.0	4.0	S	7.0	7.0	14%	
	Adult	72	4.0	10.0	A	4.0	4.0	8%	
Total		F	35.0			35.0		70%	bov
		M	15.0			15.0		30%	
		T	50.0			50.0		100%	

**Figure 4:** Part “Parameters” of interface PROJ of DYNMOD – General characteristics.

Demography			Year	1	2	3	4	5	6	7	8	9
<b>Reproduction</b>												
	Parturition rate (/year)			0.50	0.50	0.30	0.50	0.50	0.50	0.50	0.50	0.50
	Net prolificacy rate			1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	% of female at birth			50%	50%	50%	50%	50%	50%	50%	50%	50%
<b>Mortality (%)</b>												
- /age class if duration <1 year												
- /year if duration >=1 year												
	Female	J		13%	13%	45%	13%	13%	13%	13%	13%	13%
		S		5%	5%	13%	5%	5%	5%	5%	5%	5%
		A		3%	3%	21%	3%	3%	3%	3%	3%	3%
	Male	J		13%	13%	45%	13%	13%	13%	13%	13%	13%
		S		5%	5%	13%	5%	5%	5%	5%	5%	5%
		A		3%	3%	21%	3%	3%	3%	3%	3%	3%
<b>Offtake (%)</b>												
- /age class if duration <1 year												
- /year if duration >=1 year												
	Female	J		0%	0%	0%	0%	0%	0%	0%	0%	0%
		S		5%	5%	10%	5%	5%	5%	5%	5%	5%
		A		5%	5%	25%	5%	5%	5%	5%	5%	5%
	Male	J		10%	10%	10%	10%	10%	10%	10%	10%	10%
		S		20%	20%	30%	20%	20%	20%	20%	20%	20%
		A		21%	21%	50%	21%	21%	21%	21%	21%	21%
<b>Intake (Number)</b>												
	Female	J										
		S										
		A										
	Male	J										
		S										
		A										
<b>Production</b>												
<b>Live weight (kg/animal at beginning of age group)</b>												
	Female	J		20	20	20	20	20	20	20	20	20
		S		50	50	50	50	50	50	50	50	50
		A		250	250	250	250	250	250	250	250	250
	Male	J		20	20	20	20	20	20	20	20	20
		S		70	70	70	70	70	70	70	70	70
		A		300	300	300	300	300	300	300	300	300
<b>Meat</b>												
	Carcass yield (%)			47%								
<b>Financial value (/animal)</b>												
	Offtake Female	J		40,000	40,000	24,000	40,000	40,000	40,000	40,000	40,000	40,000
		S		90,000	90,000	54,000	90,000	90,000	90,000	90,000	90,000	90,000
		A		150,000	150,000	90,000	150,000	150,000	150,000	150,000	150,000	150,000
	Male	J		40,000	40,000	24,000	40,000	40,000	40,000	40,000	40,000	40,000
		S		110,000	110,000	66,000	110,000	110,000	110,000	110,000	110,000	110,000
		A		200,000	200,000	120,000	200,000	200,000	200,000	200,000	200,000	200,000
	Intake Female	J										
		S										
		A										
	Male	J										
		S										
		A										
<b>Milk</b>												
	Offtake per lactation milked (litre)			170	170	170	170	170	170	170	170	170
	% lactations milked			80%	80%	80%	80%	80%	80%	80%	80%	80%
	Milk offtake/lactation (litre)			136	136	136	136	136	136	136	136	136
<b>Skin and hides (kg/animal)</b>												
	Female	J										
		S										
		A		5.0								
	Male	J										
		S										
		A		2.0								
<b>Wool (kg/animal)</b>												
	J											
	S											
	A											
<b>Manure (kg/animal/day)</b>												
	J			0.5								
	S			0.8								
	A			1.5								

Figure 5: Part “Parameters” of interface PROJ of DYNMOD – Demography and Production.



			Year								
Feeding			1	2	3	4	5	6	7	8	9
Dry matter requirements (% of kg LW/day)											
	Female	J	2.5%								
		S	2.5%								
		A	2.5%								
	Male	J	2.5%								
		S	2.5%								
		A	2.5%								

Figure 6: Part “Parameters” of interface PROJ of DYNMOD – Feeding.

Results												
			Year									
Population			0	1	2	3	4	5	6	7	8	9
Growth rate				1.7%	1.6%	-32.0%	1.6%	1.3%	1.4%	1.4%	1.4%	1.4%
Size	Female	F		5.1%	3.8%	-39.9%	6.7%	4.7%	3.8%	3.2%	2.7%	2.3%
		M		2.7%	2.3%	-34.5%	3.1%	2.3%	2.1%	1.9%	1.8%	1.7%
		T										
	Male	J	4.5	4.7	4.8	2.5	3.0	3.2	3.3	3.4	3.5	3.5
		S	10.0	10.3	10.7	8.7	8.0	7.9	7.9	7.9	8.0	8.1
		A	20.5	20.5	20.7	13.4	13.9	14.2	14.5	14.7	14.9	15.1
	Total	J	4.0	4.4	4.6	2.3	2.9	3.1	3.2	3.2	3.3	3.3
		S	7.0	7.2	7.5	5.5	5.2	5.2	5.4	5.5	5.6	5.7
		A	4.0	4.1	4.3	2.1	2.4	2.7	2.9	3.0	3.2	3.3
	Total	F	35.0	35.6	36.2	24.6	25.0	25.3	25.7	26.0	26.4	26.8
		M	15.0	15.8	16.4	9.8	10.5	11.0	11.4	11.8	12.1	12.4
		T	50.0	51.4	52.5	34.4	35.5	36.3	37.1	37.8	38.5	39.1
Avg. size												
Structure	Global	Female	J	4.6	4.8	3.6	2.7	3.1	3.3	3.4	3.4	3.5
			S	10.2	10.5	9.7	8.4	8.0	7.9	7.9	8.0	8.1
			A	20.5	20.6	17.1	13.7	14.1	14.3	14.6	14.8	15.0
	Male	J	4.2	4.5	3.4	2.6	3.0	3.1	3.2	3.3	3.3	
		S	7.1	7.4	6.5	5.3	5.2	5.3	5.4	5.6	5.7	
		A	4.1	4.2	3.2	2.2	2.6	2.8	2.9	3.1	3.2	
	Total	F	35.3	35.9	30.3	24.8	25.2	25.5	25.8	26.2	26.6	
		M	15.4	16.1	13.0	10.2	10.7	11.2	11.6	11.9	12.2	
		T	50.7	51.9	43.3	35.1	35.9	36.7	37.4	38.1	38.8	
	Intra-sex	Female	J	9%	9%	9%	7%	9%	9%	9%	9%	9%
			S	20%	20%	20%	25%	23%	22%	21%	21%	21%
			A	41%	40%	39%	39%	39%	39%	39%	39%	39%
Male		J	8%	9%	9%	7%	8%	8%	9%	9%	9%	
		S	14%	14%	14%	16%	15%	14%	14%	15%	15%	
		A	8%	8%	8%	6%	7%	7%	8%	8%	8%	
Total		F	70%	69%	69%	71%	70%	70%	69%	69%	68%	
		M	30%	31%	31%	29%	30%	30%	31%	31%	32%	
		T	100%	100%	100%	100%	100%	100%	100%	100%	100%	
Intra-sex	Female	J	13%	13%	13%	10%	12%	13%	13%	13%	13%	
		S	29%	29%	30%	35%	32%	31%	31%	30%	30%	
		A	59%	58%	57%	55%	56%	56%	56%	56%	57%	
	Male	J	27%	28%	28%	23%	27%	28%	28%	27%	27%	
		S	47%	46%	46%	56%	49%	48%	47%	47%	46%	
		A	27%	26%	26%	21%	23%	24%	25%	26%	27%	

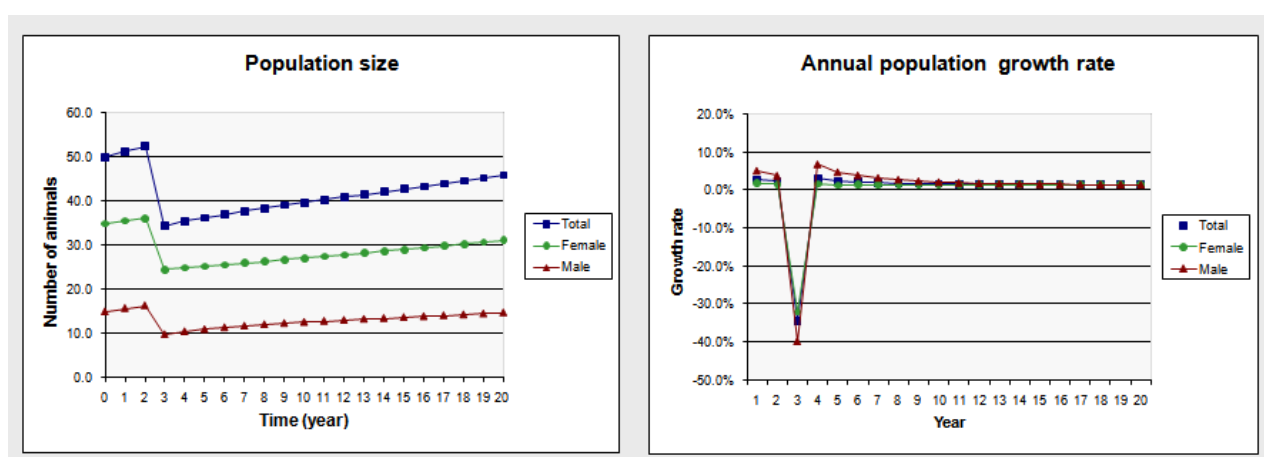
Figure 7: Part “Results” of interface PROJ of DYNMOD – Population growth rate, size and structure.

Mortality				Year								
Deaths				1	2	3	4	5	6	7	8	9
	Number	Female	J	0.6	0.7	2.0	0.4	0.4	0.5	0.5	0.5	0.5
			S	0.5	0.5	1.4	0.4	0.4	0.4	0.4	0.4	0.4
			A	0.6	0.6	3.9	0.4	0.4	0.4	0.4	0.4	0.5
		Male	J	0.6	0.6	1.9	0.4	0.4	0.4	0.4	0.4	0.5
			S	0.4	0.4	0.9	0.3	0.3	0.3	0.3	0.3	0.3
			A	0.1	0.1	0.7	0.1	0.1	0.1	0.1	0.1	0.1
	Total	F	M	1.8	1.8	7.3	1.2	1.3	1.3	1.3	1.3	
			T	1.1	1.1	3.5	0.7	0.8	0.8	0.8	0.8	
				2.8	2.9	10.8	1.9	2.0	2.1	2.1	2.2	
	Number/avg. size	Female	J	13.9%	13.9%	55.8%	14.2%	13.9%	13.8%	13.8%	13.8%	13.8%
			S	5.1%	5.1%	14.0%	5.1%	5.1%	5.1%	5.1%	5.1%	5.1%
			A	3.0%	3.0%	23.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%
		Male	J	13.9%	13.8%	55.2%	14.2%	13.8%	13.8%	13.8%	13.8%	13.8%
			S	5.0%	5.1%	13.8%	5.0%	5.1%	5.1%	5.1%	5.1%	5.1%
			A	3.0%	3.0%	22.1%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%
	Total	F	M	5.0%	5.1%	24.1%	5.0%	5.0%	5.1%	5.1%	5.1%	5.1%
			T	6.9%	7.0%	26.8%	6.9%	7.0%	7.0%	7.0%	6.9%	6.9%
				5.6%	5.7%	24.9%	5.5%	5.6%	5.7%	5.7%	5.7%	5.6%
Production				Year								
Offtake				1	2	3	4	5	6	7	8	9
	Number	Female	J	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
			S	0.5	0.6	1.1	0.4	0.4	0.4	0.4	0.4	0.4
			A	2.2	2.2	5.5	1.5	1.5	1.5	1.6	1.6	1.6
		Male	J	0.5	0.5	0.4	0.3	0.3	0.3	0.4	0.4	0.4
			S	1.6	1.7	2.5	1.2	1.2	1.2	1.2	1.3	1.3
			A	1.2	1.3	2.4	0.7	0.8	0.8	0.9	0.9	1.0
	Total	F	M	2.7	2.8	6.6	1.9	1.9	2.0	2.0	2.0	2.0
			T	3.3	3.4	5.3	2.2	2.3	2.4	2.5	2.6	2.6
				6.0	6.2	11.9	4.1	4.2	4.3	4.5	4.6	4.7
	Number/avg. size	Female	J	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
			S	5.2%	5.2%	11.4%	5.2%	5.2%	5.2%	5.2%	5.2%	5.2%
			A	10.7%	10.7%	32.1%	10.8%	10.7%	10.7%	10.7%	10.7%	10.7%
		Male	J	11.3%	11.2%	13.1%	11.5%	11.2%	11.2%	11.2%	11.2%	11.2%
			S	22.6%	22.6%	38.3%	22.5%	22.6%	22.6%	22.7%	22.7%	22.7%
			A	30.2%	30.2%	75.5%	30.3%	30.2%	30.2%	30.2%	30.2%	30.2%
	Total	F	M	7.8%	7.7%	21.7%	7.7%	7.7%	7.7%	7.7%	7.7%	7.7%
			T	21.5%	21.4%	41.0%	21.3%	21.3%	21.3%	21.4%	21.5%	21.5%
				11.9%	11.9%	27.5%	11.7%	11.7%	11.8%	11.9%	12.0%	12.0%
Intake												
Number/avg. size	Total	F	M	0%	0%	0%	0%	0%	0%	0%	0%	0%
			T	0%	0%	0%	0%	0%	0%	0%	0%	0%
				0%	0%	0%	0%	0%	0%	0%	0%	0%
SV + O - I												
Number	Total	F	M	3.3	3.3	-4.8	2.2	2.3	2.3	2.3	2.4	2.4
			T	4.1	4.0	-1.0	2.7	2.8	2.8	2.8	2.9	2.9
			%variation	7.4	7.4	-5.9	4.9	5.0	5.1	5.2	5.2	5.3
Number/size	Total	F	M									
			T	9.6%	9.4%	-13.4%	8.9%	9.0%	9.1%	9.1%	9.1%	9.1%
				27.1%	25.6%	-6.2%	27.8%	26.4%	25.6%	24.9%	24.5%	24.1%
Number/avg. size	Total	F	M									
			T	14.8%	14.3%	-11.2%	14.3%	14.2%	14.1%	14.0%	13.9%	13.8%
				9.5%	9.3%	-16.0%	8.9%	9.0%	9.0%	9.0%	9.1%	9.1%
			M	26.4%	25.1%	-7.8%	26.7%	25.8%	25.1%	24.5%	24.1%	23.8%
			T	14.6%	14.2%	-13.5%	14.1%	14.0%	13.9%	13.8%	13.8%	13.7%

**Figure 8:** Part “Results” of interface PROJ of DYNMOD – Mortality and Production (Number of animals).

Live weight equivalent (kg)										
Average living stock		9,541	9,723	8,139	6,553	6,683	6,832	6,980	7,120	7,254
Stock variation		167	196	-3,316	112	148	151	144	137	130
Offtake		1,318	1,349	2,731	873	905	936	965	991	1,014
Intake		0	0	0	0	0	0	0	0	0
SV + O - I		1,485	1,545	-584	985	1,053	1,087	1,109	1,127	1,144
%variation			4.0%	-137.8%	268.5%	6.9%	3.2%	2.0%	1.6%	1.5%
Meat equivalent (kg)										
Average living stock		4,484	4,570	3,825	3,080	3,141	3,211	3,281	3,347	3,409
Stock variation		79	92	-1,558	53	70	71	68	64	61
Offtake		619	634	1,284	410	425	440	453	466	477
Intake		0	0	0	0	0	0	0	0	0
SV + O - I		698	726	-275	463	495	511	521	530	538
%variation			4.0%	-137.8%	268.5%	6.9%	3.2%	2.0%	1.6%	1.5%
Financial equivalent										
Average living stock		5,941,188	6,059,235	3,033,677	4,058,804	4,153,672	4,252,378	4,347,049	4,436,160	4,520,200
Stock variation		112,376	123,717	-1,261,417	89,782	99,954	97,459	91,883	86,340	81,739
Offtake		819,876	839,477	1,014,272	540,148	561,367	581,593	599,992	616,547	631,506
Intake		0	0	0	0	0	0	0	0	0
SV + O - I		932,252	963,194	-247,144	629,930	661,320	679,052	691,875	702,887	713,245
%variation			3.3%	-125.7%	354.9%	5.0%	2.7%	1.9%	1.6%	1.5%
Milk (litre)										
Avg. per reproductive female		68	68	41	68	68	68	68	68	68
Total		1,395	1,401	696	930	957	975	991	1,006	1,021
%variation			0.4%	-50.3%	33.6%	2.8%	2.0%	1.6%	1.5%	1.4%
Skin & hides (kg)										
Total		13	14	32	9	9	9	10	10	10
%variation			1.0%	136.0%	-72.9%	4.4%	3.1%	2.5%	2.2%	2.0%
Wool (kg)										
Total		0	0	0	0	0	0	0	0	0
%variation										
Manure (kg)										
Total		20,116	20,496	17,061	13,713	14,058	14,383	14,686	14,971	15,242
%variation			1.9%	-16.8%	-19.6%	2.5%	2.3%	2.1%	1.9%	1.8%
Productivity measures										
Nb. new sub-adult/adult female		44%	44%	17%	44%	44%	44%	44%	44%	44%
Nb. new adult/adult female		37%	37%	11%	37%	37%	37%	37%	37%	37%
Feed		Year								
		1	2	3	4	5	6	7	8	9
Feed requirements (kg)										
DM		87,062	88,720	74,268	59,793	60,979	62,344	63,692	64,974	66,189
%variation			1.9%	-16.3%	-19.5%	2.0%	2.2%	2.2%	2.0%	1.9%

**Figure 9:** Part “Results” of interface PROJ of DYNMOD – Production (Live weight, Meat, etc.) and Feeding requirements.



**Figure 10:** Part “Graphics” of interface PROJ of DYNMOD.

# 4 Numerical examples

All the examples developed in this chapter were simplified for facilitating the presentation.

## 4.1 De-worming impact in sheep herd

In this example, we used STEADY1 and STEADY2 for simulating the potential gain obtained by farmers after de-worming a sheep herd, assuming a demographic steady state (see [4],[5],[6] for other examples with matrix models). We compared the productions of two hypothetical herds (control versus treated) and we estimated the approximate average benefit/cost ratio of the treatment that can expect the farmer.

In the treated herd, all animals of age  $>3$  months were assumed to be de-wormed. For the cost of treatment, we assumed that the unique expense for farmer was the purchase of medicines. The treatment cost per animal ( $>3$  months) and per year was set to 450 FCFA.

We carried out a simulation for each herd (control and treated). Parameters and results are presented in Figure 11 and Figure 12.

### 4.1.1 STEADY1

#### *General characteristics*

Juvenile age class duration was set to 3 months, for being able to get the number of animals of exact age  $>3$  months at steady state and to calculate the cost of de-worming. Other age class durations were set to 9 (sub-adults) and 120 (adults) months for females, and to 9 and 24 months for males. The total herd initial herd size was set to 30 animals.

#### *Demography and production*

In the control herd, parturition rate was set to 0.90/year (corresponding to an average interval between parturitions of 1.11 years) and net prolificacy rate to 1.01. Proportion of females at birth was 50%. Mortalities were the same for females and males. Lambs  $\leq 3$  months old were assumed not commercially used by farmers. Offtake rate of adult females was calibrated to get an annual herd growth rate of 0% (demographic equilibrium).

In the treated herd, de-worming was assumed to reduce the mortality of 30% in all sexes and age classes, and to increase the parturition and prolificacy rates of 5%.

Productions of each herd were assessed in numbers of animals used by farmers and in financial values.

### *Results*

The annual herd growth rate went from 0 to 7.7% after treatment. Total annual productions (i.e. stock variation + offtake) were 9.3 animals in the control herd and 12.2 animals in the treated herd. Reported to the mean herd size, this corresponded to global production rates of 31.1 and 39.1%, respectively. Total annual productions in financial value went up from 155,963 to 193,301 FCFA (relative increase of 24%). The cost of de-worming was calculated from the average size of the treated herd (sub-adults and adults):

$$C = [(4.8 + 15.9) + (4.5 + 2)] * 450 = 12,240 \text{ FCFA.}$$

In the scenario considered, the expected average benefit/cost ratio of de-worming was thus:

$$B/C = (193,301 - 155,963) / 12,240 = 3.1.$$

#### **4.1.2 STEADY2**

We used the same characteristics as in STEADY1. The constant numbers of adult females and males were set to 16 and 2, respectively (in STEADY1, they were 16 and 1.7). Parameters and results of the simulations are presented in Figure 13 and Figure 14. Total annual productions (stock variation + offtake) were 9.3 animals in the control herd and 12.1 animals in the treated herd. This corresponded to global production rates of 30.7 and 37.7%, respectively. Total annual productions in financial value went up from 154,334 to 198,760 FCFA (relative increase of 29%). Following the same calculation as before, the cost of de-worming was:

$$[(5.2 + 16.0) + (4.9 + 2.0)] * 450 = 12,645 \text{ FCFA}$$

The expected average benefit/cost ratio of de-worming was then:

$$B/C = (198,760 - 154,334) / 12,645 = 3.5.$$

In this example, STEADY1 and STEADY2 reached close results (B/C of 3.1 versus 3.5) but higher differences can be observed depending on the studies.

STEADY1										Parameters										Results									
Age classes					Population					Production					Population					Mortality									
Duration (month)		Exact age (year)			Size		Structure			Live weight (kg/animal at beginning of age group)			Ref.		Coef.	Actual	Size		Death										
		from		to	Initial		Global	Intra-sex																					
Female	Juvenile	3	0.0	0.3	Female	J	1.8	6%	8%	Female	J		1.00		0	Female	J	1.8	1.8	1.8	0.5	28.7%	28.7%						
	Sub-adult	9	0.3	1.0		S	4.5	15%	20%		S		1.00		0		S	4.5	4.5	4.5	1.2	27.6%	27.6%						
	Adult	120	1.0	11.0		A	16.0	53%	72%		A		1.00		0		A	16.0	16.0	16.0	1.7	10.4%	10.4%						
Male	Juvenile	3	0.0	0.3	Male	J	1.8	6%	23%	Male	J		1.00		0	Male	J	1.8	1.8	1.8	0.5	28.7%	28.7%						
	Sub-adult	9	0.3	1.0		S	4.2	14%	54%		S		1.00		0		S	4.2	4.2	4.2	1.1	27.4%	27.4%						
	Adult	24	1.0	3.0		A	1.7	6%	23%		A		1.00		0		A	1.7	1.7	1.7	0.2	9.4%	9.4%						
Demography					Total					Meat					Total					Total									
Reproduction					F					Carcass yield (%)					F					F									
Parturition rate (/year)					M										M					M									
Net prolificacy rate					T					Financial value (/animal)					T					T									
% of female at birth					30.0					Female					30.0					30.0									
					100%					Female					100%					100%									
					100%					J					100%					100%									
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STEADY1				Parameters				Results			
Age classes				Population				Production			
Duration (month)		Exact age (year)		Size		Structure		Live weight (kg/animal at beginning of age group)			
		from	to					Ref.		Coef.	Actual
Female	Juvenile	3	0.0	Female	J	1.9	6%	Female	J	1.00	0
	Sub-adult	9	0.3		S	4.7	16%		S	1.00	0
	Adult	120	1.0		A	15.3	51%		A	1.00	0
Male	Juvenile	3	0.0	Male	J	1.9	6%	Male	J	1.00	0
	Sub-adult	9	0.3		S	4.4	15%		S	1.00	0
	Adult	24	1.0		A	1.9	6%		A	1.00	0
Demography				Total		F	21.8	Meat		Carcass yield (%)	
Reproduction		Ref.	Coef.	M	8.2	27%	100%			0%	
Parturition rate (/year)		0.90	1.05	T	30.0	100%	100%	Financial value (/animal)		Female	
Net prolificacy rate		1.01	1.05					Female		J	5,000
% of female at birth		50%	1.00					S		9,000	1.00
								A		12,000	1.00
Mortality (%)				Feeding		Dry matter requirements (% of kg LW/day)		Male		J	5,000
- /age class if duration <1 year						Ref.	Coef.	S		12,000	1.00
- /year if duration >=1 year				Female		J	1.00	S		23,000	1.00
Female		J	7%	S		S	1.00	A		5,000	1.00
S		19%	0.70	A		A	1.00	Male		J	5,000
A		10%	0.70	Male		J	1.00	S		12,000	1.00
Male		J	7%	S		S	1.00	A		23,000	1.00
S		19%	0.70	A		A	1.00	Milk		Offtake per lactation milked (litre)	
A		10%	0.70	Total		F	21.8	% lactations milked		1.00	
				M		8.2	27%	Offtake per lactation (litre)		0.0	
				T		30.0	100%			0	
Offtake (%)				Type				Skin and hides (kg/animal)		Female	
- /age class if duration <1 year								Female		J	1.00
- /year if duration >=1 year				OV				S		1.00	0.0
Female		J	0%	Male		J	1.00	A		1.00	0.0
S		7%	1.00	S		S	1.00	Male		J	1.00
A		17%	1.00	A		A	1.00	S		1.00	0.0
Male		J	0%	Total		F	21.8	A		1.00	0.0
S		20%	1.00			M	8.2	Wool (kg/animal)		J	
A		90%	1.00			T	30.0	J		1.00	0.0
Population growth rate (%)								S		1.00	0.0
								A		1.00	0.0
								Manure (kg/animal/day)		J	
								J		1.00	0.0
								S		1.00	0.0
								A		1.00	0.0
								Financial equivalent		Avg liv. stock	
								Avg liv. stock		354,169	Nb. new sub-adult/adult f.
								Offtake		167,111	Nb. new adult/adult f.
								SV + Offtake		193,301	
								Feeding			
								Feed requirements (kg)			
								Dry matter		0	

STEADY2					Parameters					Results											
Age classes					Population				Production			Population				Mortality					
Duration (month)		Exact age (year)			Size		Structure		Live weight (kg/animal at beginning of age group)			Size		Death							
		from	to		Initial	Global	Intra-sex		Ref.	Coef.	Actual				Number	Nb./size	Nb./avg size				
Female	Juvenile	3	0.0	0.3	J	1.8	6%	8%	Female	J	1.00	0	Female	J	1.8	1.8	1.8				
	Sub-adult	9	0.3	1.0	S	4.5	15%	20%		S	1.00	0		S	4.5	4.5	4.5				
	Adult	120	1.0	11.0	A	16.0	53%	72%		A	1.00	0		A	16.0	16.0	16.0				
Male	Juvenile	3	0.0	0.3	J	1.8	6%	22%	Male	J	1.00	0	Male	J	1.8	1.8	1.8				
	Sub-adult	9	0.3	1.0	S	4.2	14%	52%		S	1.00	0		S	4.2	4.2	4.2				
	Adult	24	1.0	3.0	A	2.0	7%	25%		A	1.00	0		A	2.0	2.0	2.0				
Demography					Feeding				Meat			Production				Mortality					
Reproduction		Ref.	Coef.	Actual	Dry matter requirements (% of kg LW/day)				Financial value (/animal)			Stock variation + Offtake									
Parturition rate (/year)		0.90	1.00	0.90	Ref.			Coef.	Actual	Female			J	5,000	1.00	5000	Number		Nb./size	Nb./avg size	
Net prolificacy rate		1.01	1.00	1.01	Female			J	1.00	0.0%	S			9,000	1.00	9000	S		0.0%	0.0%	
% of female at birth		50%	1.00	50%	S			1.00	0.0%	A			12,000	1.00	12000	A		0.0%	0.0%		
					Male			J	1.00	0.0%	Male			J	5,000	1.00	5000	J		0.0%	0.0%
					S			1.00	0.0%	S			12,000	1.00	12000	S		0.0%	0.0%		
					A			1.00	0.0%	A			23,000	1.00	23000	A		0.0%	0.0%		
Mortality (%)					Milk			Offtake			Live weight equivalent (kg)				Milk (litre)						
- /age class if duration <1 year					Offtake per lactation milked (litre)			1.00			Avg liv. stock				Avg. per reprod. fem.						
- /year if duration >=1 year					% lactations milked			1.00			Offtake				Total						
					Offtake per lactation (litre)			0			SV + Offtake										
Female					J	7%	1.00	7%	Skin and hides (kg/animal)			Meat equivalent (kg)				Skin & hides (kg)					
S					19%	1.00	19%	Female			J	1.00	0.0	Avg liv. stock				Wool (kg)			
A					10%	1.00	10%	S			1.00	0.0	Offtake				Manure (kg)				
Male					J	7%	1.00	7%	Male			J	1.00	0.0	SV + Offtake						
S					19%	1.00	19%	A			1.00	0.0	SV + Offtake								
A					10%	1.00	10%	Wool (kg/animal)			Financial equivalent				Productivity measures						
					Type			OV			Avg liv. stock				Nb. new sub-adult/adult f.						
											SV + Offtake				Nb. new adult/adult f.						
											SV + Offtake										
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STEADY2					Parameters					Results																	
Age classes					Population			Production		Population			Mortality														
Duration (month)		Exact age (year)			Size		Structure		Live weight (kg/animal at beginning of age group)			Size		Death													
		from	to				Initial	Global	Intra-sex	Ref.	Coef.	Actual			Number	Nb./size	Nb./avg size										
Female	Juvenile	3	0.0	0.3	Female	J	2.0	6%	9%	Female	J	1.00	0	Female	J	2.0	2.0	2.0	0.3	13.2%	13.2%						
	Sub-adult	9	0.3	1.0		S	5.2	16%	22%		S	1.00	0		S	5.2	5.2	5.2	1.0	18.8%	18.8%						
	Adult	120	1.0	11.0		A	16.0	50%	69%		A	1.00	0		A	16.0	16.0	16.0	1.2	7.2%	7.2%						
Male	Juvenile	3	0.0	0.3	Male	J	2.0	6%	22%	Male	J	1.00	0	Male	J	2.0	2.0	2.0	0.3	13.2%	13.2%						
	Sub-adult	9	0.3	1.0		S	4.9	15%	55%		S	1.00	0		S	4.9	4.9	4.9	0.9	18.6%	18.6%						
	Adult	24	1.0	3.0		A	2.0	6%	23%		A	1.00	0		A	2.0	2.0	2.0	0.1	7.2%	7.2%						
Demography					Total		F	23.2	72%	100%	Meat			Total		F	23.2	23.2	23.2	2.4	10.3%	10.3%					
Reproduction					M		8.8	28%	100%	Carcass yield (%)			M		8.8	8.8	8.8	1.3	14.8%	14.8%							
Parturition rate (/year)					T		32.0	100%					T		32.0	32.0	32.0	3.7	11.6%	11.6%							
Net prolificacy rate										Financial value (/animal)																	
% of female at birth										Female			J	5,000	1.00	5000	Production										
										S			9,000	1.00	9000	Offtake											
										A			12,000	1.00	12000	Stock variation + Offtake											
																Number			Nb./size	Nb./avg size	Number	Nb./size	Nb./avg size				
																Female			J	0.0	0.0%	0.0%	0.0	0.0%	0.0%		
																S			0.5	10.3%	10.3%	0.5	10.3%	10.3%			
																A			5.0	31.0%	31.0%	5.0	31.0%	31.0%			
																Male			J	0.0	0.0%	0.0%	0.0	0.0%	0.0%		
																S			1.5	31.4%	31.4%	1.5	31.4%	31.4%			
																A			5.1	252.6%	252.6%	5.1	252.6%	252.6%			
																Total			F	5.5	23.7%	23.7%	5.5	23.7%	23.7%		
																M			6.6	74.5%	74.5%	6.6	74.5%	74.5%			
																T			12.1	37.7%	37.7%	12.1	37.7%	37.7%			
																			Live weight equivalent (kg)			Milk (litre)					
																			Avg liv. stock			0	Avg. per reprod. fem.			0	
																			Offtake			0	Total			0	
																			SV + Offtake			0					
																			Meat equivalent (kg)			Skin & hides (kg)			0		
																			Avg liv. stock			0	Wool (kg)			0	
																			Offtake			0					
																			SV + Offtake			0	Manure (kg)			0	
																						Financial equivalent			Productivity measures		
																			Avg liv. stock			362,680	Nb. new sub-adult/adult f.			95%	
																			Offtake			198,760	Nb. new adult/adult f.			83%	
																			SV + Offtake			198,760					
																			Feeding								
																			Feed requirements (kg)								
																			Dry matter			0					
</																											

## 4.2 Drought impact on a cattle population

In this example, we used PROJ for simulating the impact of a drought on the dynamics of a cattle population in the Sahel. Parameters were assumed to be constant over the simulation period except during the year of drought.

Parameters for normal years were defined from literature. Those for drought were assumed to be estimated from a field survey implemented after the shock (e.g., using a cross-sectional 12-month retrospective method, [7]). We assumed that this survey showed a general increase in mortalities (particularly for young and old animals) and offtake (particularly for adult males), and a drop in calving. Parameters and the results of the simulation are those presented in Figure 4 to Figure 10 of Chapter 2.

### 4.2.1 Parameters

#### *General characteristics*

The projection was carried out over 20 years, with the drought simulated in year “3”. The female age class durations were set to 12, 36 and 132 months for juveniles, sub-adults and adults, respectively. Male age class durations were set to 12, 36 and 72 months. Corresponding age classes (in exact ages) are “0 to 1 year”, “>1 to 4 years” and “>4 to 15 years” for females and “0 to 1 year”, “>1 to 4 years” and “>4 to 10 years” for males.

We standardized the initial total population size to 50 animals. The assumed population sex-by-age structure is presented in Figure 4 (females represented 70% of the initial stock).

#### *Demography and production*

Parturition rate was set to 0.50/year on normal years (corresponding to an average interval between parturitions of 2.22 years) and to 0.30/year in the drought year. The net prolificacy rate was unchanged during the projection and set to 1. The proportion of females at birth was 50%.

No massive restocking was assumed after the drought. Offtake probabilities represented “net offtake” (balance between offtake and intake). Mortalities and offtake used in the projection are presented in Table 1.

The milk offtake per milked lactation was set to 170 litres and the proportion of lactations milked (within all lactations) to 80%. The live weights and net offtake financial values in normal and drought years are presented in Table 2. We assumed a drop of 40% of the selling prices during the drought.

The carcass yield was set to 47% and the daily feed requirements in dry matter to 2.5% of the live weight for all sexes and age classes.

**Table 1:** Mortality and offtake probabilities (%) used in interface PROJ to simulate the drought impact in the 20-year projection.

Sex	Age class	Probabilities (%)			
		Normal year		Drought	
		Mortality <sup>(a)</sup>	Offtake	Mortality <sup>(a)</sup>	Offtake
Female	Juvenile	13	0	45	0
	Sub-adult	5	5	13	10
	Adult	3	5	21	25
Male	Juvenile	13	10	45	10
	Sub-adult	5	20	13	30
	Adult	3	21	21	50

(a) Assuming no offtake (intrinsic mortality)

**Table 2:** Live weights and financial values (net offtake) used in interface PROJ to simulate the drought impact in the 20-year projection. Live weights correspond to the beginning of the age classes.

Sex	Age class	Live weight (kg)		Financial value (FCFA)	
		Normal	Drought	Normal	Drought
Female	Juvenile	20	20	40000	24000
	Sub-adult	50	50	90000	54000
	Adult	250	250	150000	90000
Male	Juvenile	20	20	40000	24000
	Sub-adult	70	70	110000	66000
	Adult	300	300	200000	120000

## Results

The drought generated a drop of -32, -40 and -35% in female, male and total population sizes.

After the drought, the annual population growth rate reached 3.1% and then regularly converged to 1.4%. Based on the offtake rates considered in normal years, the stock was not fully reconstituted at the end of the simulation period (Figure 10). In normal years, female, male and global production rates represented almost 9, 23 and 14% of the mean population sizes.

Dynamics of the other productions and of the feed requirements are presented in Figure 9. For a more complete evaluation of the drought impact, the user should carry out a projection without the drought and compare the productions with the results presented here. After the shock, a faster restocking could also be simulated by decreasing the offtake rates (assuming that farmers will tend to keep their stock) or/and by importing a large number of females to boost the reproduction and population renewal.

# 5 Appendix

## 5.1 The demographic model

### 5.1.1 Introduction

DYNMOD uses a discrete-time matrix model {for theory of matrix models, see [1],[8-11]}. A matrix model consists on a system of mathematical equations whose parameters are the demographic rates (reproduction, mortality, etc). Equations simulate the evolution of the population size (by sex and age class) on regular and successive time intervals  $(t-1, t)$ . It can be written in a matrix form:

$$x_t = A_{t-1} * x_{t-1},$$

where:

- $x_{t-1}$  and  $x_t$  represent population state vectors whose components are the numbers of the animals present respectively at time  $t-1$  and time  $t$  in the different sexes and age classes;
- $A_{t-1}$  represents a projection matrix, containing the demographic rates for the time interval between  $t - 1$  and  $t$ .

Numerous applications of matrix models can be found in literature on tropical livestock populations (e.g. [5],[12-23]).

### 5.1.2 Equations

#### *Dynamics over a month*

This section presents the mathematical equations used in DYNMOD for simulating the dynamics over a given month. For simplification, we only present equations for females (the principle is almost the same for males although the model is a little more complex). The population state vector  $x_t$  corresponding to the female life cycle represented in Figure 1 is a vector with 5 age-class components:

$$x_t = \begin{bmatrix} x_t(1) \\ x_t(2) \\ x_t(3) \\ x_t(4) \\ x_t(5) \end{bmatrix} \begin{matrix} \text{Births} \\ \text{Juveniles (without births)} \\ \text{Sub - adults} \\ \text{Adults (without final culling)} \\ \text{Final culling} \end{matrix},$$

where  $x_t(i)$  is the number of females of age class  $i$  living in the population at time  $t$ . In interfaces, results of age classes 1 and 2 are aggregated into category “Juveniles” (including births). Results of age classes 4 and 5 are aggregated into category “Adults” (including final culled animals). The dynamics over the month are given by the matrix equation:

$$x_t = FEC * (I - CULL) * G * (I - DEA - OFF) * x_{t-1},$$

where  $DEA$ ,  $OFF$ ,  $G$ ,  $CULL$ ,  $I$  and  $FEC$  are 5\*5 matrices defined as follows.

$I$  is the identity matrix:

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

$DEA$  and  $OFF$  are the mortality and offtake matrix:

$$DEA = \begin{bmatrix} p_{dea,1} & 0 & 0 & 0 & 0 \\ 0 & p_{dea,2} & 0 & 0 & 0 \\ 0 & 0 & p_{dea,3} & 0 & 0 \\ 0 & 0 & 0 & p_{dea,4} & 0 \\ 0 & 0 & 0 & 0 & p_{dea,5} \end{bmatrix},$$

$$OFF = \begin{bmatrix} p_{off,1} & 0 & 0 & 0 & 0 \\ 0 & p_{off,2} & 0 & 0 & 0 \\ 0 & 0 & p_{off,3} & 0 & 0 \\ 0 & 0 & 0 & p_{off,4} & 0 \\ 0 & 0 & 0 & 0 & p_{off,5} \end{bmatrix},$$

where  $p_{dea,i}$  and  $p_{off,i}$  are the monthly mortality and offtake probabilities for females in age class  $i$ . Probabilities  $p_{dea,2}$  to  $p_{dea,4}$  and  $p_{off,2}$  to  $p_{off,4}$  are calculated from the age-class or annual probabilities  $P_{dea,c}$  and  $P_{off}$  entered in DYNMOD. Probabilities for birth class are set equal to juveniles ( $p_{dea,1} = p_{dea,2}$  and  $p_{off,1} = p_{off,2}$ ) and probabilities for culling class are set equal to adults ( $p_{dea,5} = p_{dea,4}$  and  $p_{off,5} = p_{off,4}$ ).

The vector representing the death numbers is calculated by  $DEA * x_{t-1}$ . The one representing the offtake numbers by  $OFF * x_{t-1}$  on which is added the final culling (see below).

$(I - DEA - OFF)$  represents the survival matrix  $S$ :

$$S = \begin{bmatrix} s_1 & 0 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 & 0 \\ 0 & 0 & s_3 & 0 & 0 \\ 0 & 0 & 0 & s_4 & 0 \\ 0 & 0 & 0 & 0 & s_5 \end{bmatrix},$$

where  $s_i$  is the monthly surviving probability for animals in age class  $i$  ( $s_i = 1 - p_{dea,i} - p_{off,i}$ ).

$G$  is the between age-classes transition matrix:

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1-g_2 & 0 & 0 & 0 \\ 0 & g_2 & 1-g_3 & 0 & 0 \\ 0 & 0 & g_3 & 1-g_4 & 0 \\ 0 & 0 & 0 & g_4 & 1 \end{bmatrix},$$

where  $g_i$  is the probability of animals to grow up from age class  $i$  to age class  $i+1$  after surviving until the end of the month. In DYNMOD, this probability is calculated by ([1]):

$$g_i = \frac{(s_i)^{d_{i-1}} - (s_i)^{d_i}}{1 - (s_i)^{d_i}},$$

where  $d_i$  is the duration of age class  $i$ .

$CULL$  is the final culling matrix:

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The vector representing the culled animals numbers (final culling) is calculated by  $CULL * G * (I - DEA - OFF) * x_{t-1}$  and is added to the vector of offtake numbers.

Finally,  $FEC$  is the fecundity matrix:

$$F = \begin{bmatrix} 1 & 0 & 0 & f_4 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where  $f_4$  is the monthly number of female births generated per adult female:

$$f_4 = \text{proportion of females at birth} * \text{net prolificacy rate} * h_{\text{par}} / 12,$$

(with  $h_{\text{par}}$  being the annual parturition rate entered in DYNMOD).

#### *Maximal or constant sizes*

When maximal or constant sizes are specified in PROJ or STEADY2, respectively, DYNMOD begins by calculating the vector  $(I - CULL) * G * (I - DEA - OFF) * x_{t-1}$ , and then compare it to the specified sizes. The eventual surplus is removed from the population and added to the offtake vector.

#### *Steady state*

In STEADY1, the steady state sex-by-age structure is calculated by the dominant right eigenvector (standardized to 100%) of matrix  $A = FEC * (I - CULL) * G * (I - DEA - OFF)$ , and the steady state annual growth rate by  $\lambda^{12}$  where  $\lambda$  is the dominant eigenvalue of  $A$ .

#### *Calculation of the mortality and offtake probabilities*

In DYMODO, the entered probabilities of (intrinsic) mortality  $P_{\text{dea},c}$  and offtake  $P_{\text{off}}$  correspond to:

- the age class if the age class duration <12 months;
- to a 1-year period if the age class duration >=12 months.

Internally, DYNMOD uses monthly probabilities  $p_{\text{dea}}$  and  $p_{\text{off}}$  (instead of using directly  $P_{\text{dea},c}$  and  $P_{\text{off}}$ ) to remove biases due to interferences generated by the competing risks mortality and offtake ([24],[25],[4],[26]). These biases become negligible when the duration of the time-interval used for computations is sufficiently small, and monthly intervals are generally sufficient for ruminant livestock.

Monthly probabilities  $p_{dea}$  and  $p_{off}$  are calculated as follows. We note  $d$  the number of months corresponding to  $P_{dea,c}$  and  $P_{off}$ . From  $P_{dea,c}$ , we calculate the instantaneous mortality hazard rate  $h_{dea}$  (/month) with the equation:

$$P_{dea,c} = 1 - \exp(-h_{dea} * d) \Rightarrow h_{dea} = \frac{-\log(1 - P_{dea,c})}{d}.$$

Knowing  $P_{dea,c}$ ,  $P_{off}$  and  $h_{dea}$ , we then calculate the instantaneous offtake hazard rate  $h_{dea}$  (/month) with the equation:

$$P_{off} = \frac{h_{off}}{h_{dea} + h_{off}} [1 - \exp(-(h_{dea} + h_{off}) * d)].$$

At this step, a Newton-Raphson algorithm is used because the above equation is non-linear and  $h_{off}$  cannot be directly separated from the other variables. Finally, the monthly probabilities  $p_{dea}$  and  $p_{off}$  are calculated by using both following equations:

$$p_{dea} = \frac{h_{dea}}{h_{dea} + h_{off}} [1 - \exp(-(h_{dea} + h_{off}))],$$

$$p_{off} = \frac{h_{off}}{h_{dea} + h_{off}} [1 - \exp(-(h_{dea} + h_{off}))].$$



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